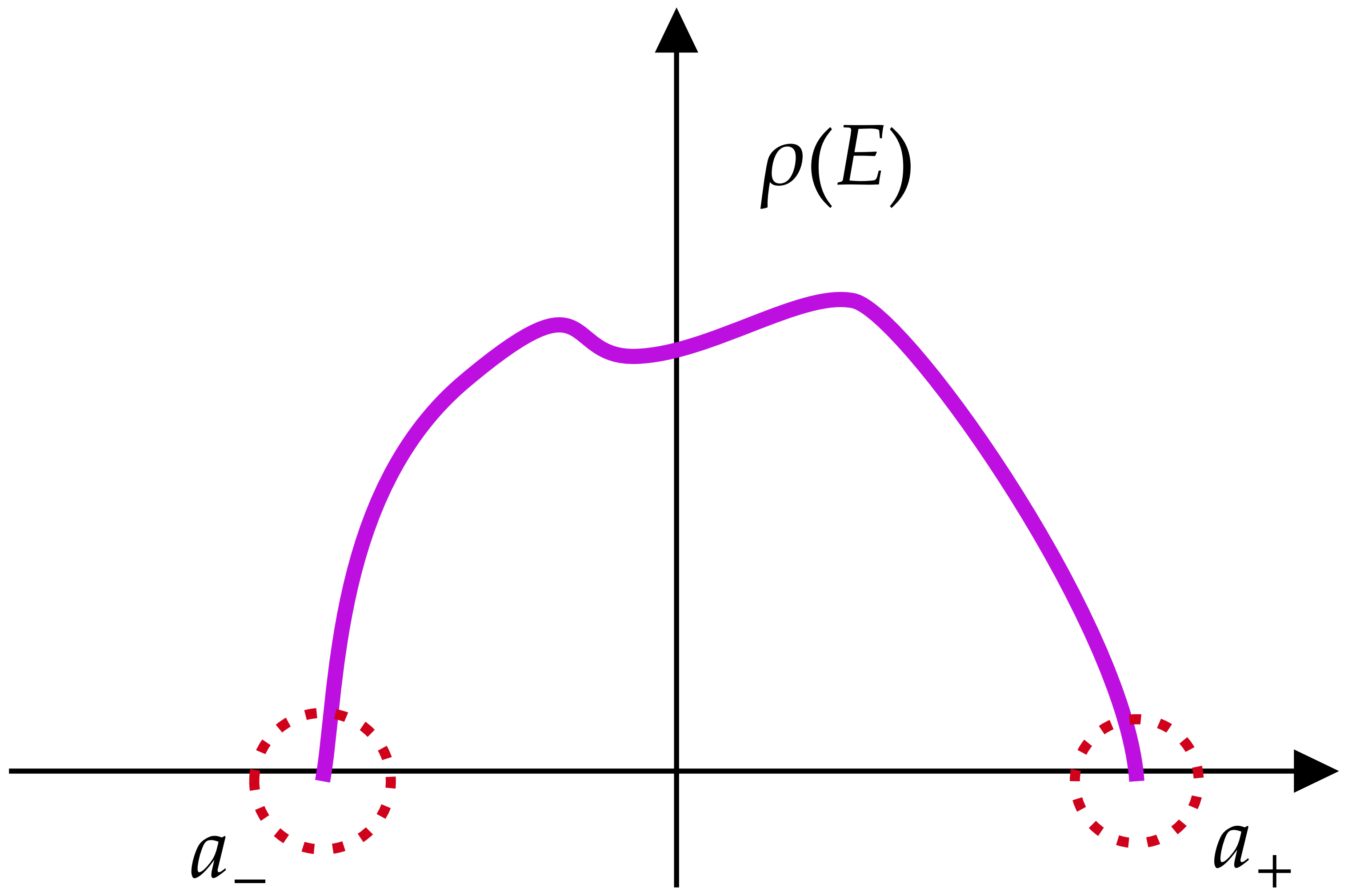
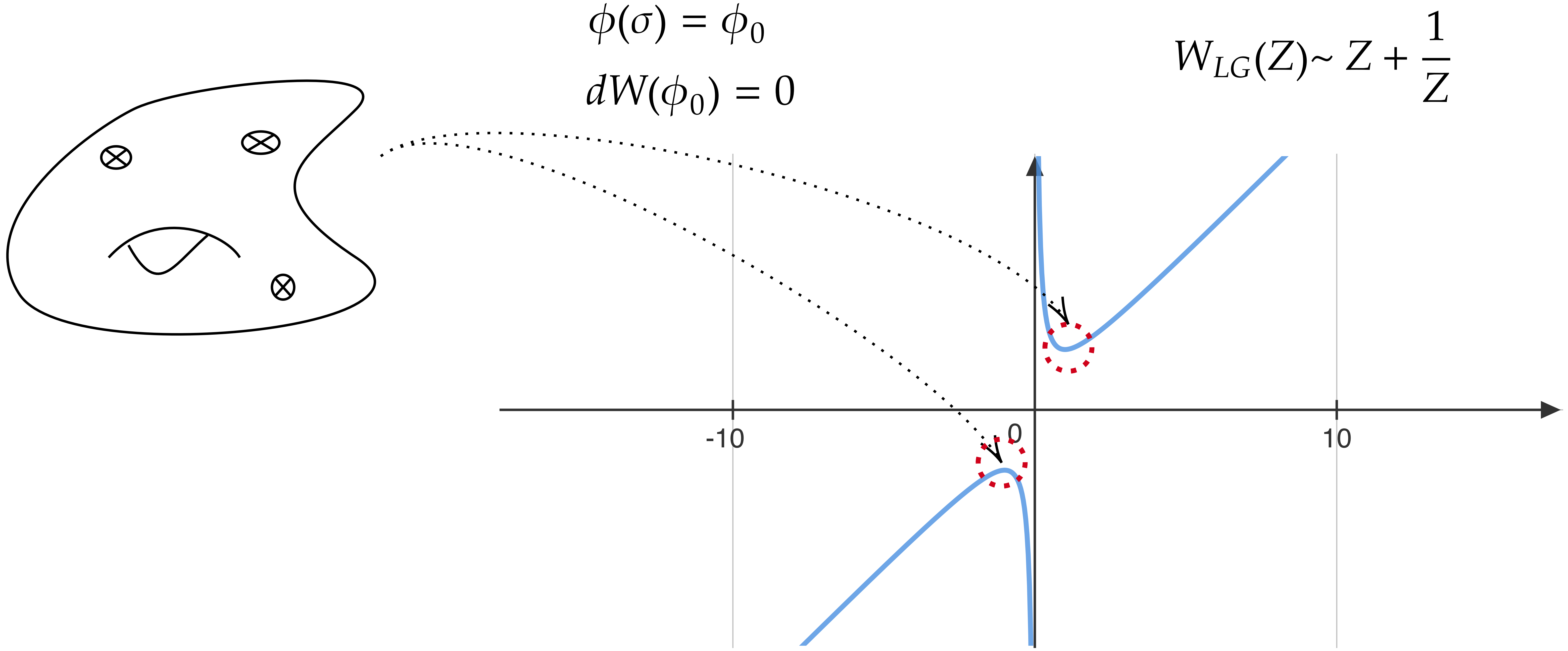
Worldsheet Duals to 1-Matrix Models





arXiv: 2604.03126 + WIP , w/ A. Giacchetto (ETH) & R. Gopakumar (ICTS)

Edward Mazenc (ETH Zürich) - May. 20, 2026

Meeting at the Extremes Workshop, IAS

The Big Motivating Question

Large N gauge theories can secretly be (closed) string theories.
(e.g. AdS/CFT, Gopakumar/Vafa Conifold, QCD String…)

The Big Motivating Question

Large N gauge theories can secretly be (closed) string theories.
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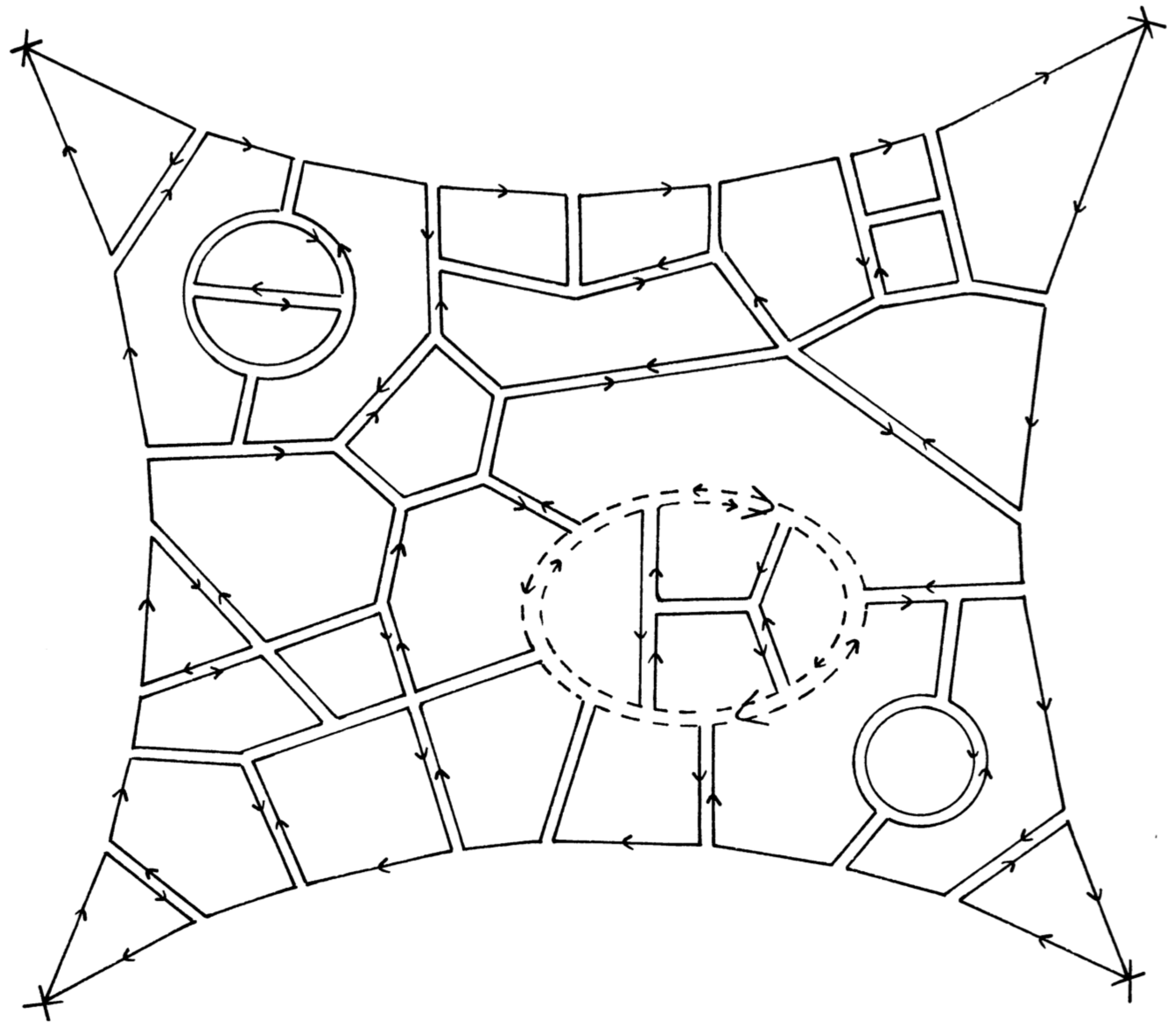
Can we derive the string dual
to a gauge theory?

't Hooft's *Picture*

Large N expansion looks like genus expansion of strings



- Figure 3 -

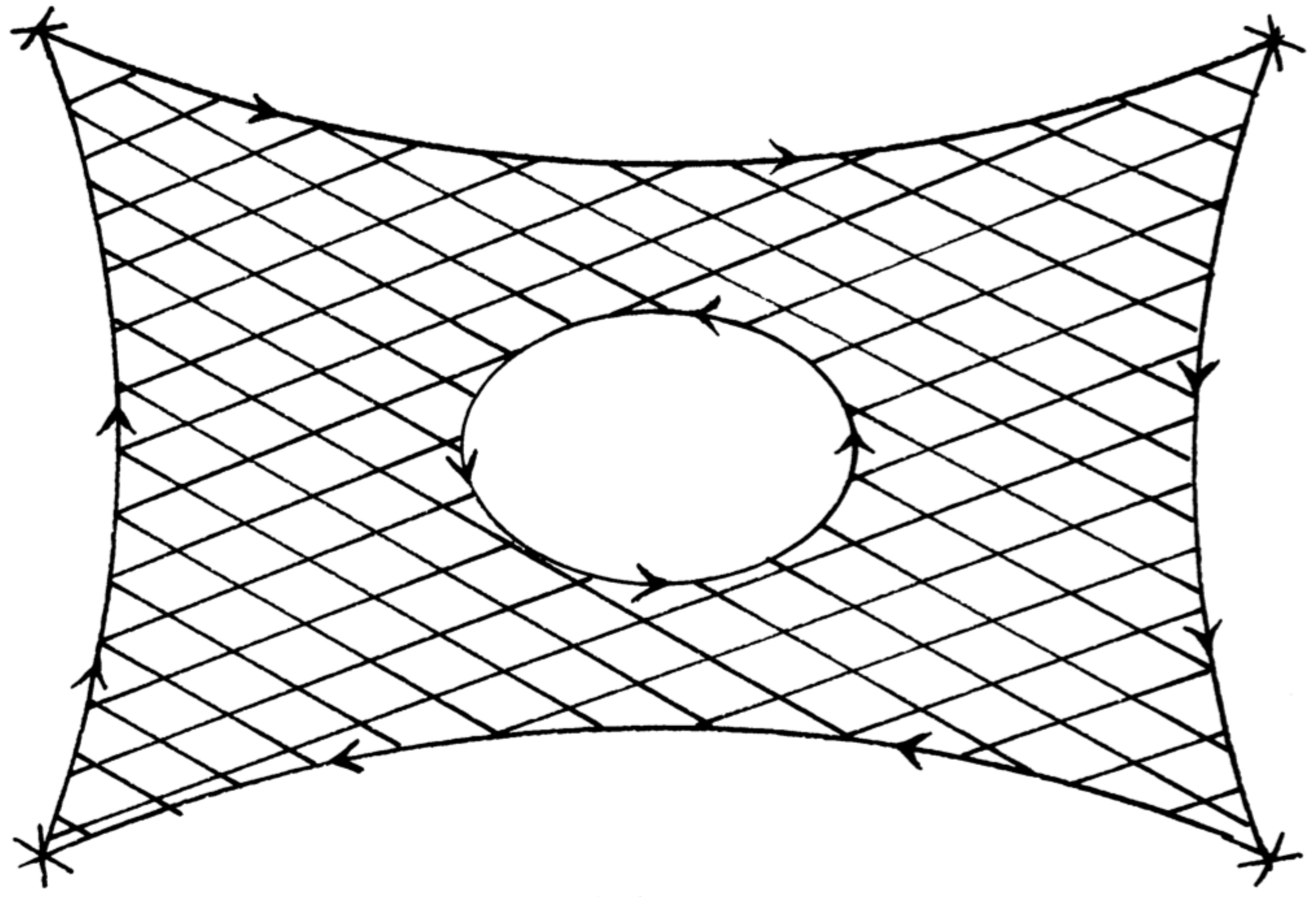
A PLANAR DIAGRAM THEORY FOR STRONG INTERACTIONS

G. 't Hooft

CERN - Geneva

Ref.TH.1786-CERN

13 December 1973



(a)

Job of String: Resum $\lambda = g_{YM}^2 N$ expansion at each order in $1/N$

An Old (Extreme) Playground

Matrix Integrals
= 0-dim Gauge theories

$$Z_N(\lambda) = \int dM_{N\times N}\, e^{-NTr\left(\frac{M^2}{2}+\frac{\lambda}{4}M^4\right)}$$

't Hooft Expansion

$$N^{-E+F}(N\lambda)^V = N^{2-2g}\lambda^V \qquad logZ_N(\lambda) = \sum_g N^{2g-2}f_g(\lambda)$$

Planar Diagrams

E. Brézin, C. Itzykson, G. Parisi*, and J. B. Zuber

Service de Physique Théorique, Centre d'Études Nucléaires de Saclay, F-91190 Gif-sur-Yvette, France

Communications in Mathematical Physics

An Old Playground

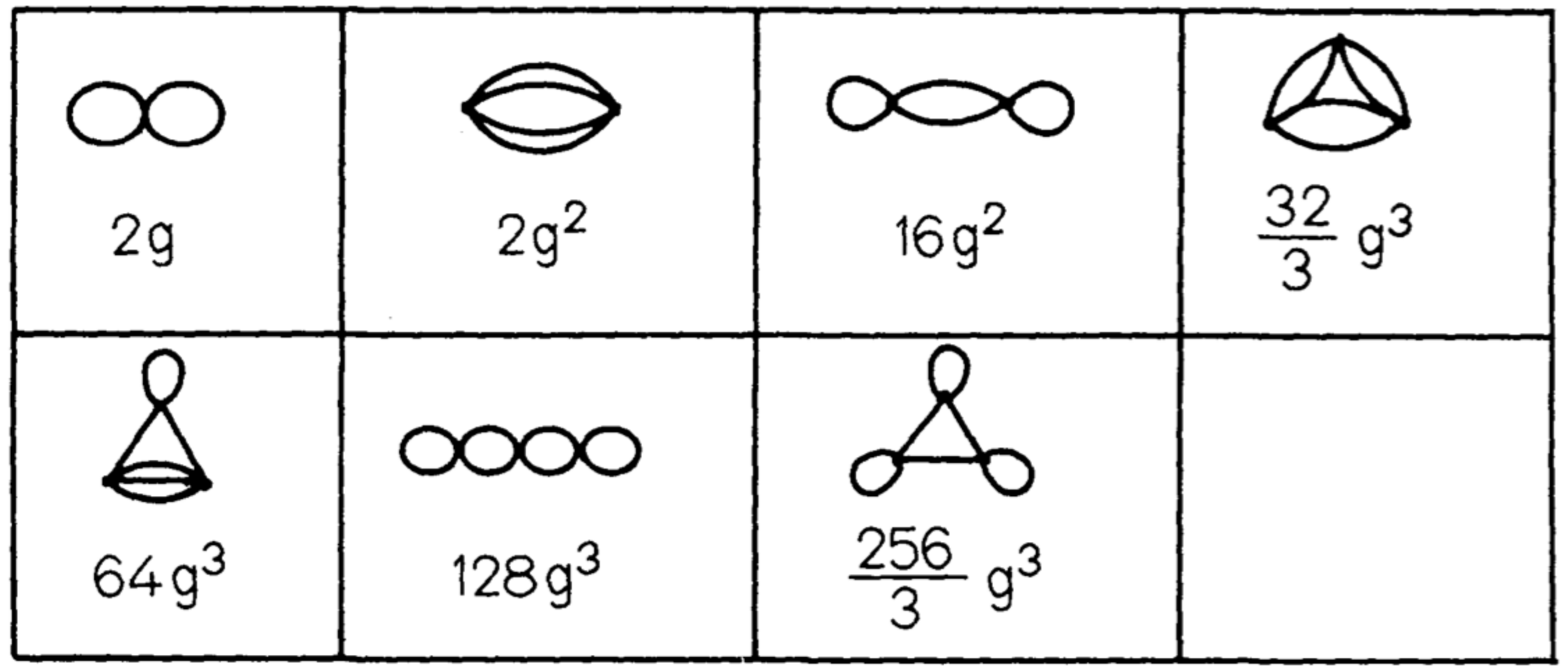
Matrix Integrals
= 0-dim Gauge theories

$$Z_N(\lambda) = \int dM_{N\times N}\, e^{-NTr\left(\frac{M^2}{2} + \frac{t_4}{4}M^4\right)}$$

't Hooft Expansion

$$N^{-E+F}(Nt_4)^V = N^{2-2g}t_4^V \qquad log\ Z_N(t_4) = \sum_{g\geq 0} N^{2-2g} f_g(t_4)$$

Table 1. Counting rules for the vacuum amplitude $E^{(0)}(g)$ in the planar limit, up to order three



$2g$	$2g^2$	$16g^2$	$\frac{32}{3}g^3$
$64g^3$	$128g^3$	$\frac{256}{3}g^3$	

[Brézin, Itzykson, Parisi, Zuber $('78)$ N.B. : $g_{them} = t_4$]

$$F_0(t_4) - F_0(t_4 = 0) =$$

$$\frac{1}{24}(a(t_4)^2 - 1)(9 - a(t_4)^2) - \frac{1}{2}\log(a(t_4)^2) =$$

$$2t_4 - 18t_4^2 + 288t_4^3 + \ldots$$

An Old Playground

Matrix Integrals
= 0-dim Gauge theories

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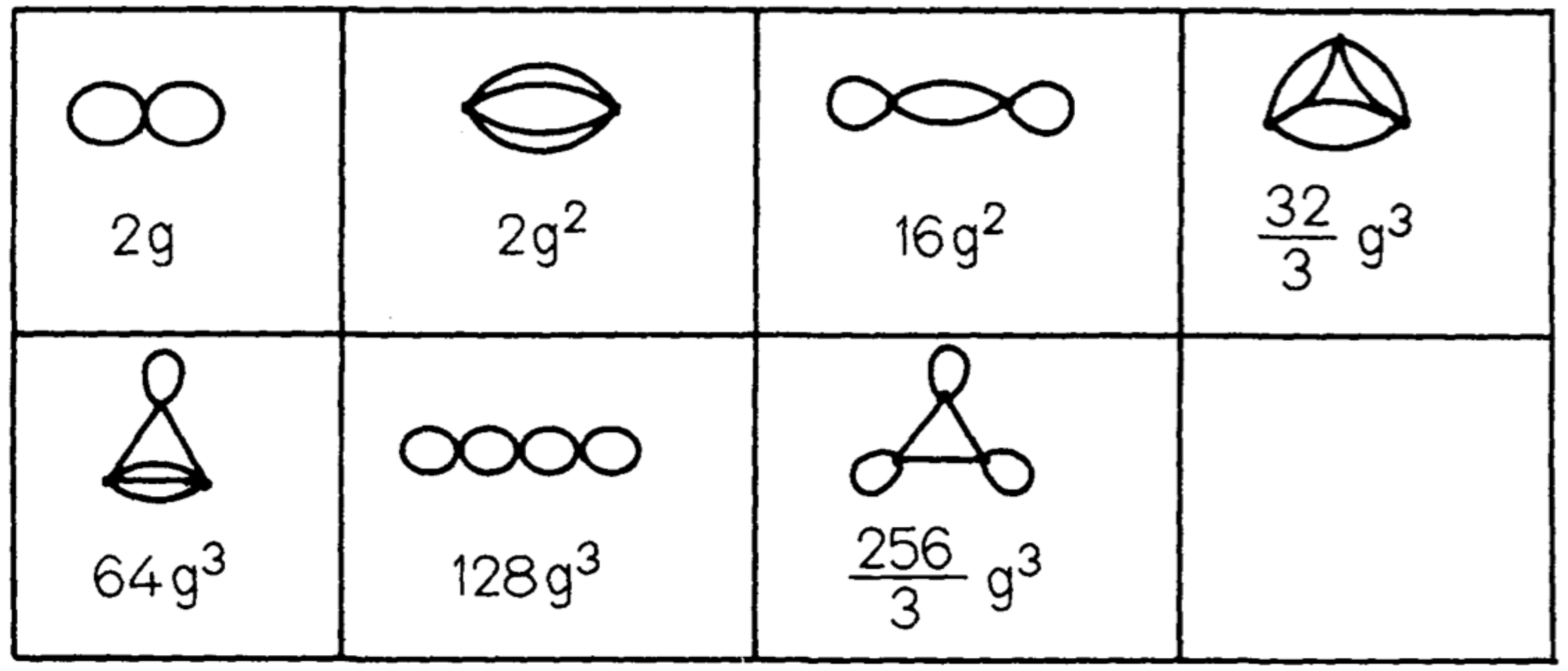

$2g$	$2g^2$	$16g^2$	$\frac{32}{3}g^3$
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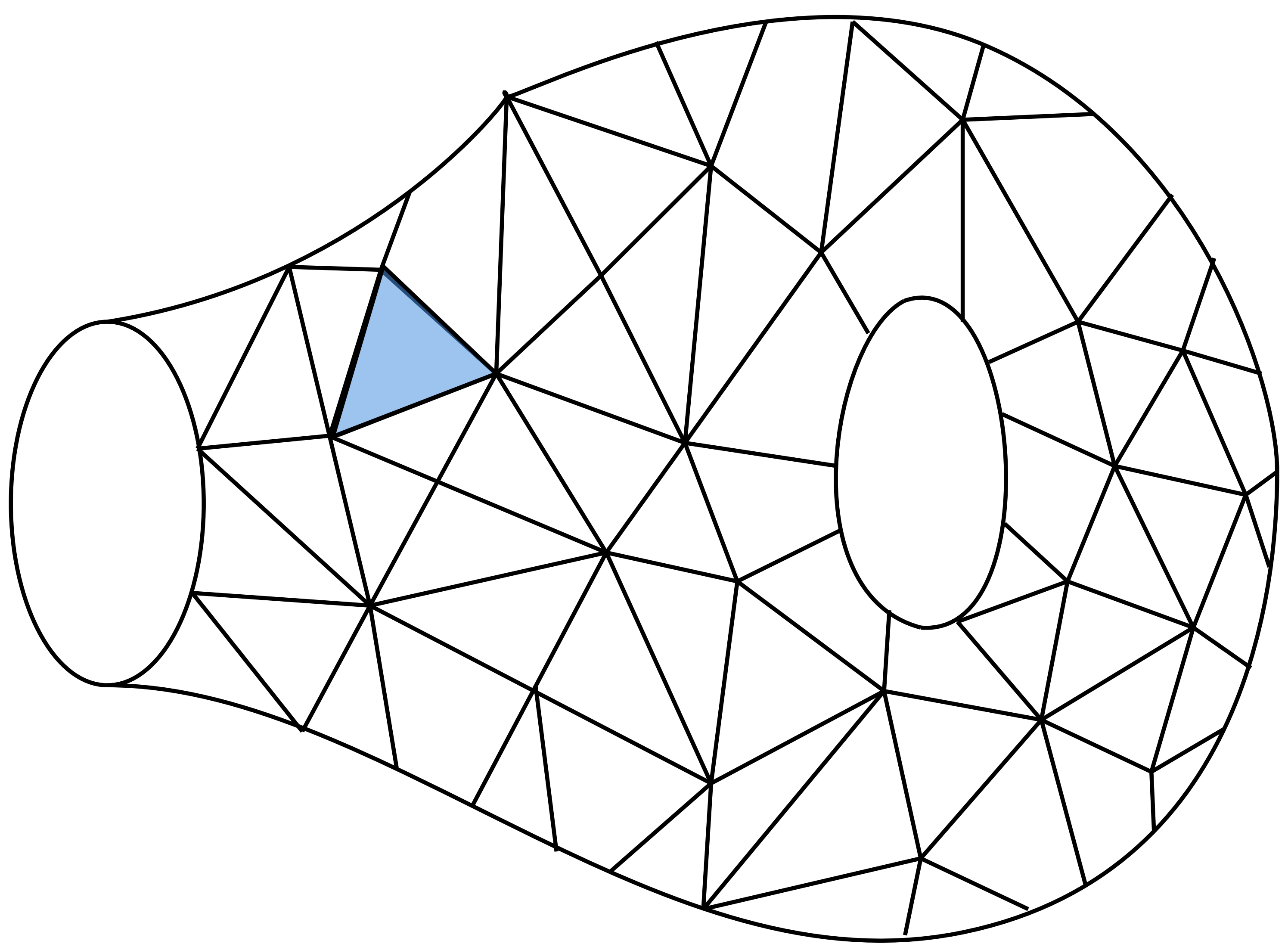
What is the string dual of such a simple gauge theory?

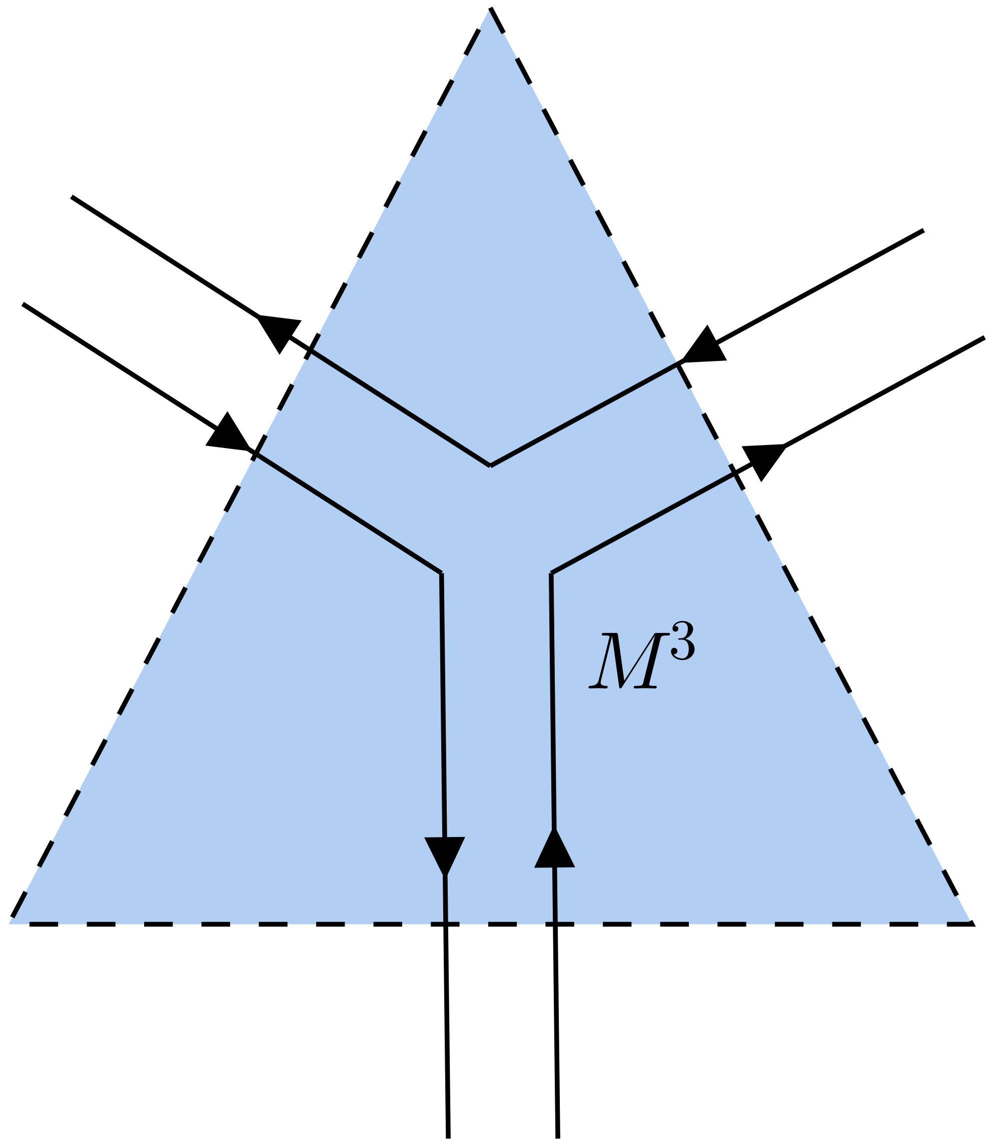
[Brézin, Itzykson, Parisi, Zuber ('78) N.B. : $g_{them} = \lambda_{here}$]

A Past Ideological Detour

(Polyakov, Gross, Migdal, Brézin, Kazakov, Kostov, Shenker, Douglas…)

Feynman Diagrams look like *discretised* worldsheets

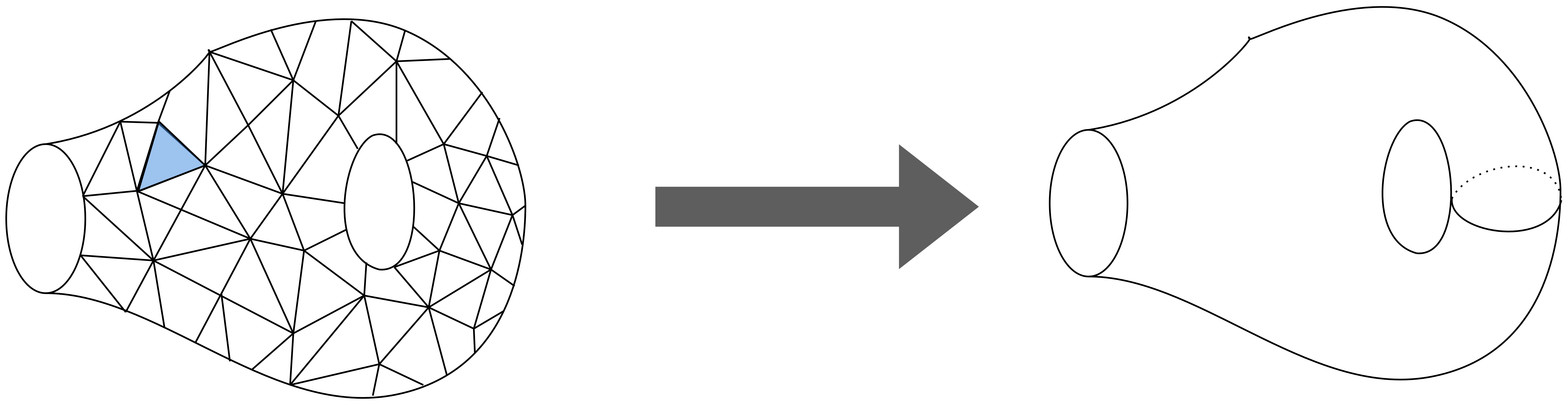




Graph dual of Feynman diagram = triangulation of worldsheet

Unlike AdS/CFT

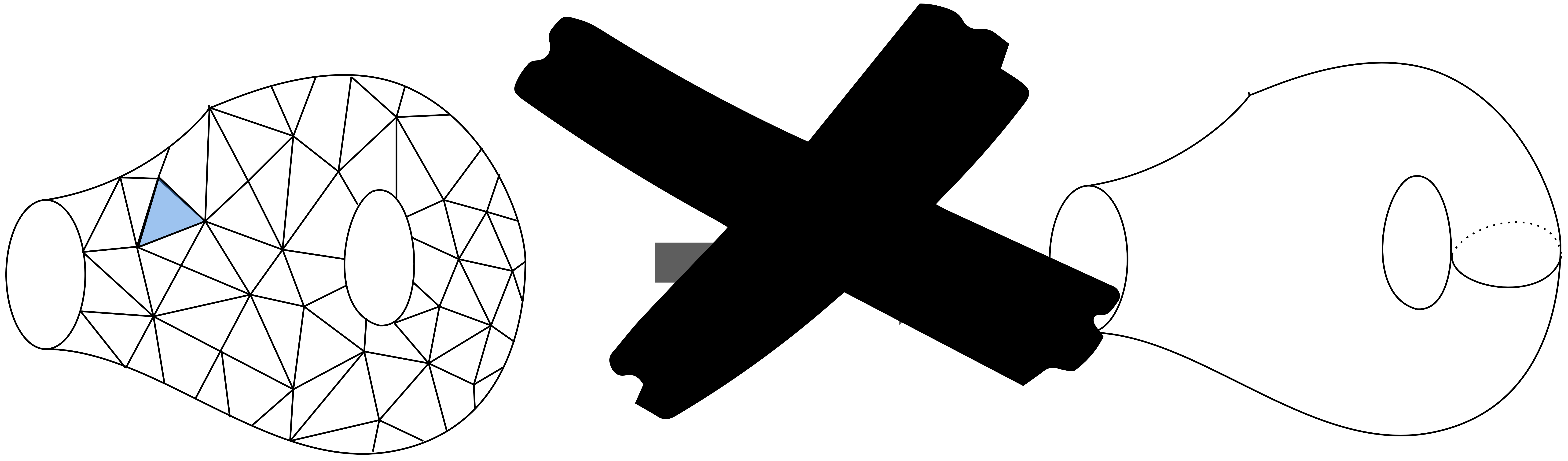
Requires continuum limit on WS/ “ double-scaling” limit on Matrix Model
(*NOT* standard ’t Hooft limit)



“Strings emerge only in limit
of (infinitely) dense Feynman Diagrams”?

Unlike AdS/CFT

Requires continuum limit on WS/ " double-scaling" limit on Matrix Model (*NOT* standard 't Hooft limit)



A Different Picture: map individual Feynman diagrams to specific worldsheets via Strebel Differentials

[Strebel ('84); Gopakumar ('03,04,05); Aharony, Komargodski, Razamat ('06); Razamat ('09); Gopakumar, Kaushik, Komatsu, EAM, Sarkar ('24)]

Beyond Perturbation Theory

A Different Picture: map individual Feynman diagrams to specific worldsheets via Strebel Differentials

Inherently perturbative in the 't Hooft coupling!

Early Motivation for Dual String:
geometrically resums the 't Hooft coupling expansion, better description at strong coupling!

Purpose of Today's Talk

Standard ’t Hooft Limit (as in AdS/CFT)

$$\int dM_{N\times N}\; e^{-N\,TrV(M)}$$

$$\boxed{\text{Provide a closed string dual to the generic } \textit{interacting} \text{ matrix model}}$$

to *all orders* in 1/N and *exactly* in the ‘t Hooft coupling(s)!

(I.e. not using Feynman diagrams)

A First Operational Goal

« *Gaiotto's Challenge* » ('25)

The Categorical 't Hooft Expansion

The general expectation is that the 't Hooft expansion of any similar QFT coincides with the perturbative expansion of a String Theory [46, 4, 61], i.e. the $F_g(\lambda, N\hbar)$ coefficients can be recast as

$$F_g(\lambda, N\hbar) = \int_{\mathcal{M}_{g,0}} \omega_g(\lambda, N\hbar) \tag{1.4}$$

for a collection of closed top forms $\omega_g(\lambda, N\hbar)$ on the moduli space $\mathcal{M}_{g,0}$ of genus g Riemann surfaces, computed as partition functions of a dual two-dimensional "worldsheet" QFT, which is topological in a cohomological sense (dg-TQFT).[2] More generally, correlation functions of "single-trace operators" such as

$$\int dM e^{-\frac{1}{\hbar}\mathrm{Tr}\left(M^2+\lambda M^4\right)} \prod_{i=1}^{n} \hbar^{-1}\mathrm{Tr}M^{k_i} \tag{1.5}$$

should arise from n-point correlation functions of local operators in the dg-TQFT, integrated over the moduli space $\mathcal{M}_{g,n}$ of Riemann surfaces with n punctures. See [44, 43, 42] and references therein for recent attempts to make this statement precise for Gaussian/free models.

At the moment, there is no systematic strategy to recover the worldsheet dg-TQFT from the data of the original QFT. The dual dg-TQFT is (conjecturally) known for a collection of examples, some of

What does that even mean?

« Gaiotto's Challenge » ('25)

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We will find a systematic way to compute the integrand on moduli space

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \langle \langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \rangle \rangle_{WS,g}$$

Finite 't Hooft coupling(s)!

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \langle \langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \rangle \rangle_{WS,g}$$

Can formulate for any large N saddle of any 1-matrix integral (single trace potential)

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$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathcal{V}_{k_i} \rangle\rangle_{WS,g}$$

Includes subtle contact terms on WS

Explicitly computable!

Our Main Result

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Many checks -
now automated as a computer code

https://cocalc.com/agiacche/ws-duals-1-mm/demo

Our Main Result

A Worldsheet Theory & Integrals over Moduli Space

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An Example:

$$\langle TrM^{2k} \rangle_{g=1}^{quartic} = \int_{\overline{\mathscr{M}}_{1,1}} \left(\frac{16k-1}{8\gamma}\psi_1 - \frac{3(1-38t_4\gamma^2)}{8\gamma(1-6t_4\gamma^2)}\kappa_1 - \frac{1}{8\gamma}[\partial\mathscr{M}_{1,1}] \right) \frac{\gamma^{2k-1}}{1-6t_4\gamma^2} \frac{(2k)!}{(k-1)!k!}$$

$$= \left(\frac{k-1}{12} - t_4\gamma^2 \frac{k-2}{2} \right) \frac{\gamma^{2k-2}}{1-12t_4} \frac{(2k)!}{(k-1)!k!}$$

$$\gamma \equiv \frac{1-\sqrt{1-12t_4}}{6t_4}$$

Our Main Result

Explicit Worldsheet Theory & Computable Intgerals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \rangle\rangle_{WS,g}$$

Dual String: ***B-twisted LG model coupled to 2d topological gravity***

Matter Theory

~ akin to ghost system

[Witten ('88) ; Vafa ('90); Verlinde^2 ('92)]

Our Main Result

Explicit Worldsheet Theory & Computable Intgerals over Moduli Space

$$\langle \prod_{i=1}^{n} Tr(M^{k_i}) \rangle_g = \int_{\bar{\mathscr{M}}_{g,n}} \langle\langle \prod_{i=1}^{n} \mathscr{V}_{k_i} \rangle\rangle_{WS,g}$$

Dual String: ***B-twisted LG model coupled to 2d topological gravity***

« matter primaries »

$$\mathcal{O}_\alpha \otimes \psi^k$$

« gravitational descendants »
(2k-forms on moduli space)

[Witten ('88); Vafa ('90); Verlinde[2] ('92); Witten ('92) Hanany, Oz, Plesser ('94); Ghoshal, Vafa('95)]

Our Main Result

Explicit Worldsheet Theory & Computable Intgerals over Moduli Space

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Dual String: ***B-twisted LG model coupled to 2d topological gravity***

Quasi-Universal Operator Dictionary:

$$TrM^k \leftrightarrow \mathscr{V}_k \equiv \sum_{\alpha \in +,-} \sum_{d=1}^{k-1} c^k_{\alpha,d}\, \mathscr{O}_\alpha \otimes \psi^d$$

Our Main Result

For concreteness: 1-cut, even potential

Quasi-Universal Operator Dictionary:

$$TrM^k \leftrightarrow \mathscr{V}_k \equiv \sum_{\alpha\in +,-} \sum_{d=1}^{k-1} c^k_{\alpha,d}\, \mathcal{O}_\alpha \otimes \psi^d$$

Width of eigenvalue distribution

$$c^k_{\alpha,d} = \frac{\gamma(\alpha\gamma)^{k-d-1}}{\Omega_\alpha} \frac{k!}{\lfloor \frac{k-d-1}{2} \rfloor ! \lceil \frac{k-d-1}{2} \rceil !}$$

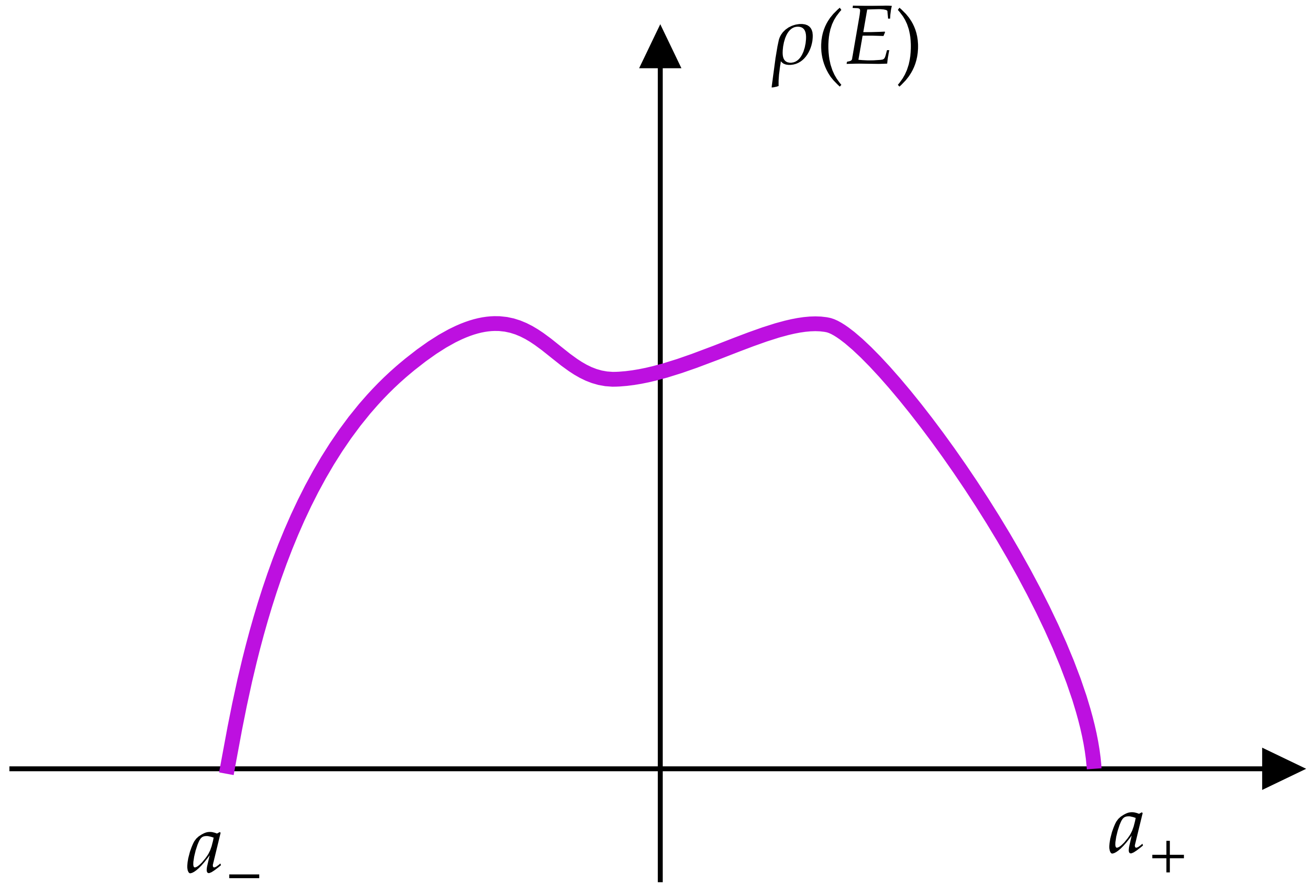
~ controls square-root vanishing of eigenvalue distribution

From the Matrix Model to the WS

Matrix Model Spectral Curve Data

$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)} \longrightarrow y^2 = V'(x)^2 + P(x)$$

$$y = \frac{V'(x)}{2} - \langle Tr \left(\frac{1}{x-M} \right) \rangle_{g=0}$$



$$x(z) = \gamma \left(z + \frac{1}{z} \right) + \delta$$ ***(Quasi) Universal***

$$y(z) = \sum_k u_k z^k$$ ***Depends on $V(M)$***

From the Matrix Model to the WS

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$$0 = \int dM \frac{\partial}{\partial M_{ij}} \left(\frac{1}{x - M} \right)_{ji} e^{-NTrV(M)}$$

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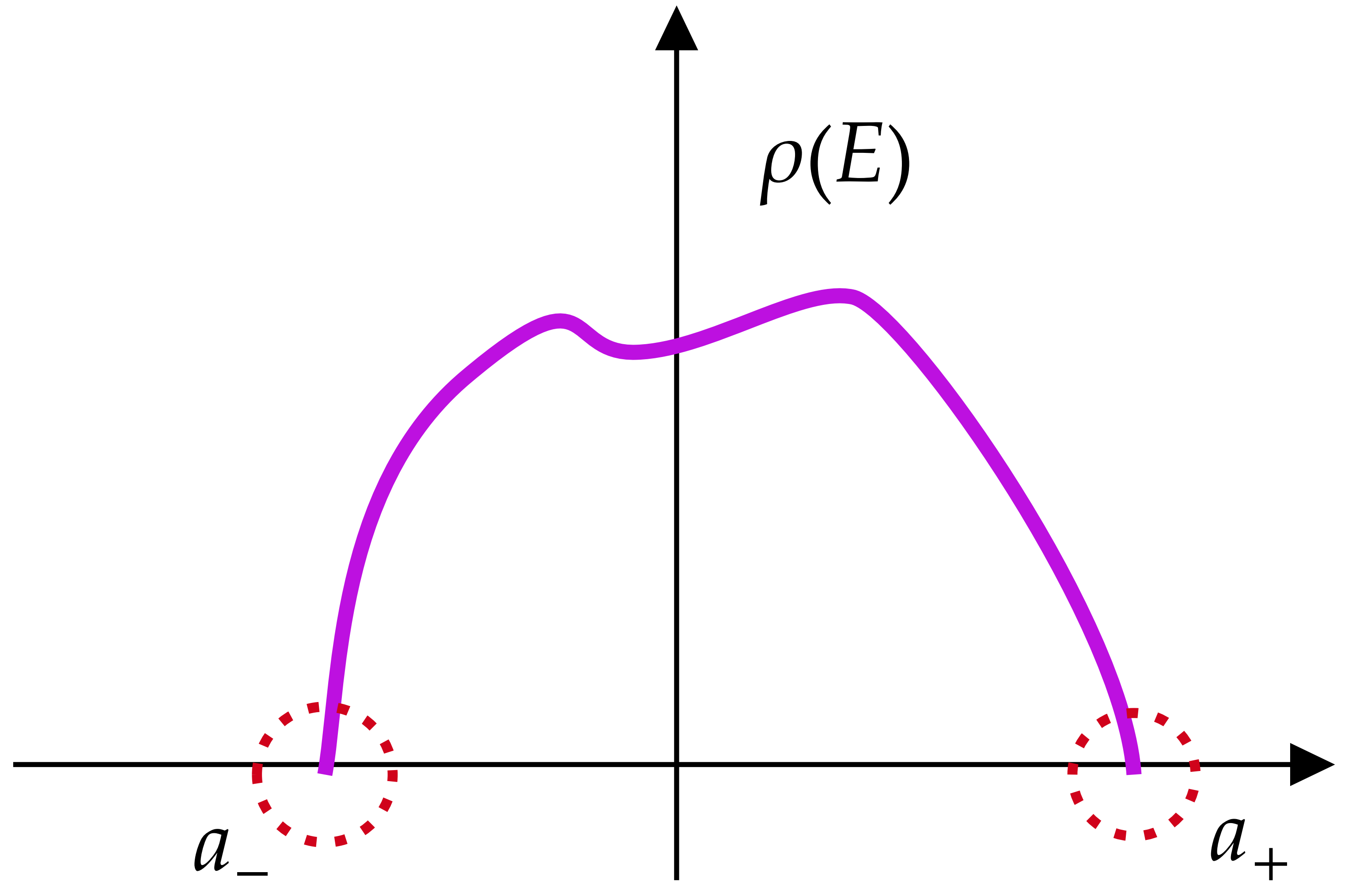


$$x(z) = \gamma \left(z + \frac{1}{z} \right) + \delta$$

(Quasi) Universal

$$dx(z) = 0 \leftrightarrow z = \pm 1$$

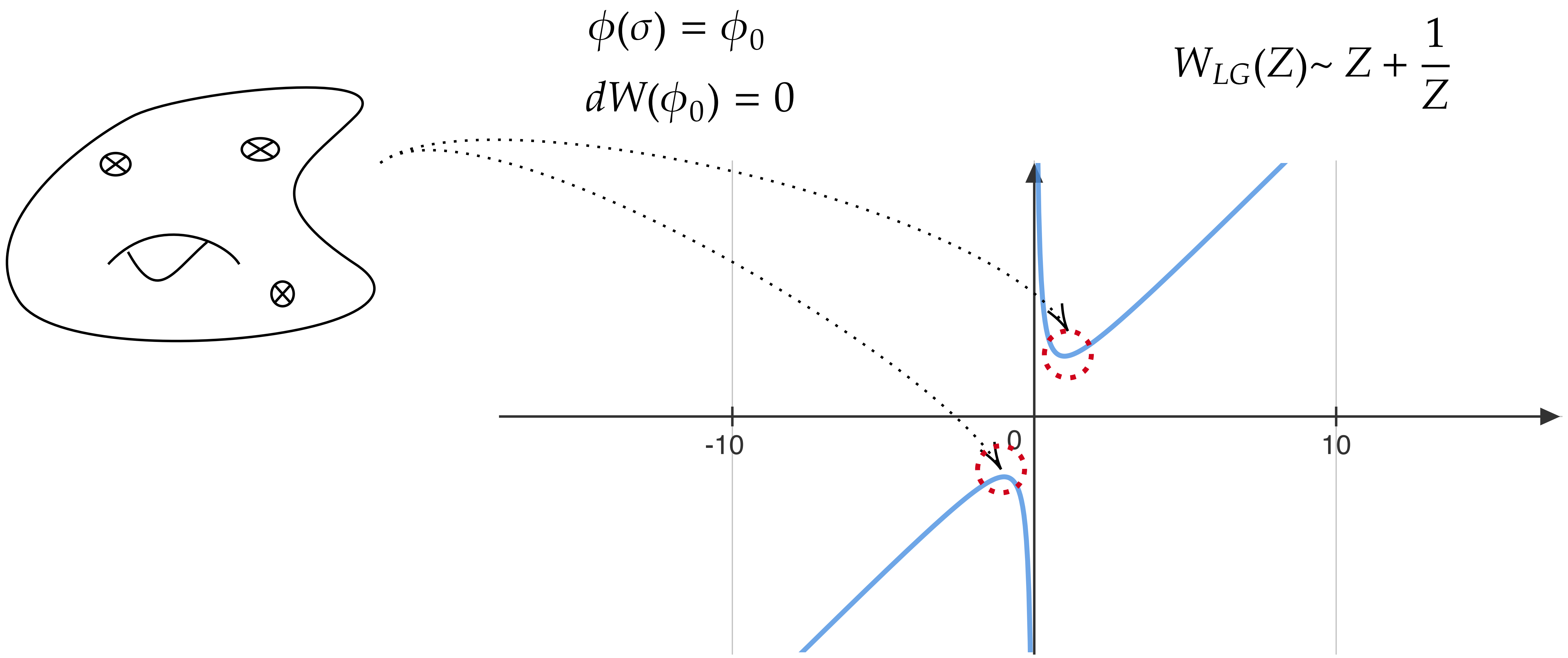
Zeroes of dx

The WS Matter Theory

B-Twisted LG Models (Vafa, '90)

$$S_{LG} = \int_{WS} d^2\sigma\ G_{\phi\bar{\phi}} |\partial\phi|^2 + |\partial_\phi W(\phi)|^2 + \rho\bar{\partial}\chi + \bar{\rho}\partial\bar{\chi} + \rho\partial_\phi^2 W\bar{\rho} + \bar{\chi}\partial_{\bar{\phi}}^2 \bar{W}\chi$$

Path Integral Localizes!



WS gets mapped into critical points of LG potential

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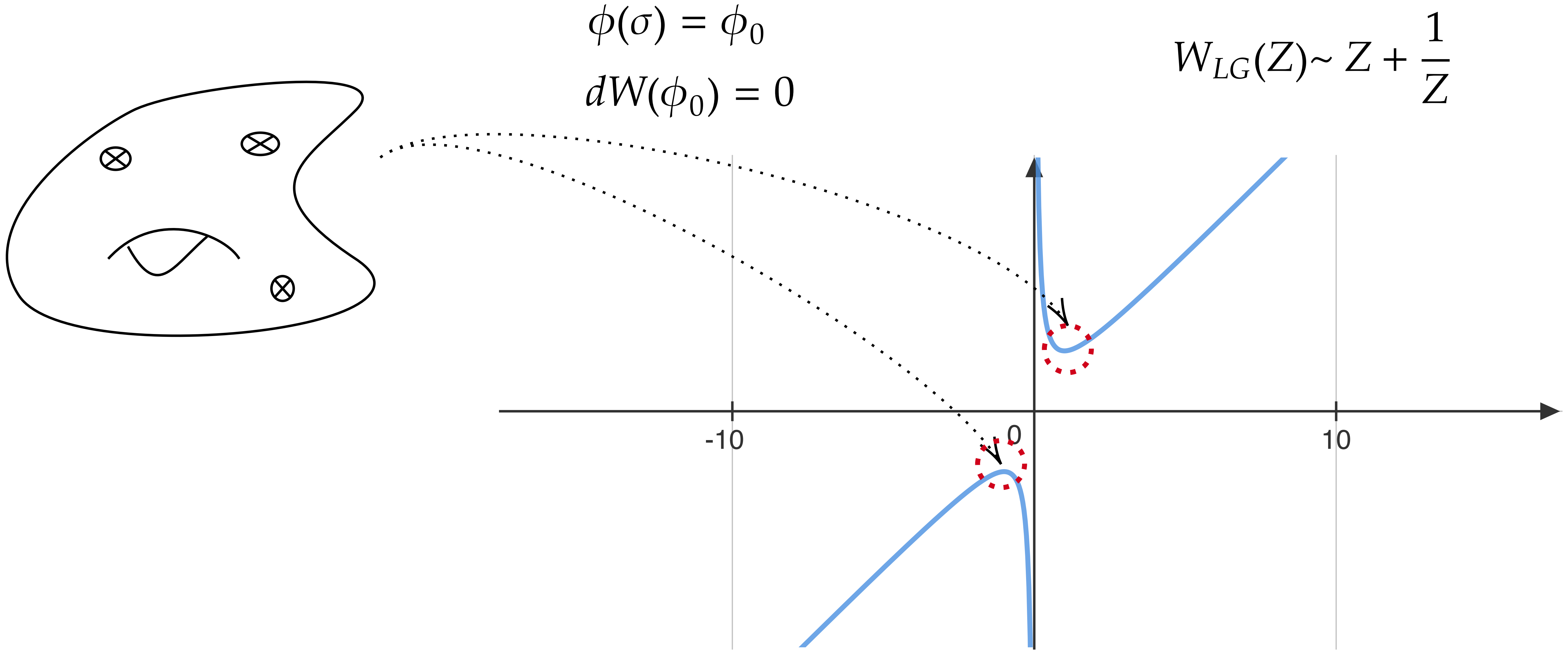

(Twisted) Matter Theory is essentially a TFT!

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Dictionary: $W(\phi) \leftrightarrow x(z)$ $\qquad \Omega = dy$

Our Relation to Matrix Model Spectral Curve

The WS Matter Theory

B-Twisted LG Models (Vafa, '90)

$$S_{LG} = \int_{WS} d^2\sigma \; G_{\phi\bar{\phi}} \,|\, \partial\phi \,|^2 + |\, \partial_\phi W(\phi) \,|^2 + \rho\bar{\partial}\chi + \bar{\rho}\partial\bar{\chi} + \rho\partial^2_\phi W \bar{\rho} + \bar{\chi}\partial^2_{\bar{\phi}}\bar{W}\chi$$

Dictionary: $W(\phi) \leftrightarrow x(z)$ $\quad \Omega = dy$

Quasi-Universal:

$$W = \gamma(\phi + 1/\phi) + \delta$$

Details of $V(M)$= geometry of target

Pure Matter Theory Correlators

Chiral Ring (BRST)

$$\frac{\mathbb{C}[\phi]}{dW}$$

Ring Relation

$$d(\phi + \frac{1}{\phi}) = 0 \leftrightarrow \phi^2 = 1$$

Phys. Ops

$$\{1, \phi\}$$

Can compute from path integral localization

$$\langle \phi^{l_1}(\sigma_1) \ldots \phi^{l_n}(\sigma_n) \rangle_g = \phi^{l_1} \ldots \phi^{l_n} \left(\frac{\partial^2 W}{\partial y(\phi)^2} \right)^{g-1} \Big|_{\phi = \pm 1}$$

[Witten ('88); Lerche, Warner, Vafa ('89); Hori, Vafa ('00)]

Towards our Op. Dictionary

$$\mathcal{O}_+ = \frac{1+\phi}{2}$$

Diagonal Basis Phys. Ops,
Supported on each critical point

$$\phi_{crit} = \pm 1$$

$$\mathcal{O}_- = \frac{1-\phi}{2}$$

$$\langle \mathcal{O}_\pm^n \rangle_g = H_\pm^{g-1}$$

$$H_\pm = \frac{\nabla dW}{\Omega^2}\bigg|_\pm = \frac{\partial^2 x}{\partial y^2}\bigg|_\pm$$

Can show correlators diagonal at all genus for TFT

Comparing w/ Matrix Model

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \int_{\bar{\mathscr{M}}_{g=0,n=3}} \langle \prod_{i=1}^{n} \mathscr{V}_{2k_i} \rangle_{WS,g}$$

$$\mathscr{V}_{2k} = \sum_{\alpha \in +,-} \sum_{d=1}^{2k-1} c_{\alpha,d}^{2k} \, \mathscr{O}_{\alpha} \otimes \psi^{d}$$

Can ignore gravity

$\mathscr{M}_{0,3} \sim pt$

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \langle \mathscr{O}_{+}^{3} \rangle_{g=0} \prod_{i=1}^{3} c_{+,0}^{(2k_i)} + \langle \mathscr{O}_{-}^{3} \rangle_{g=0} \prod_{i=1}^{3} c_{-,0}^{(2k_i)}$$

Universal Answer for Planar 3pt Function from LG model

Example: Quartic MM

[Gopakumar,Pius ('14)]

MM perturbation theory

$$\langle \prod_{i=1}^{3} Tr(M^{2k_i}) \rangle_{g=0} = \sum_{s=0}^{\infty} \frac{(3t_4)^s}{s\,!} \frac{(k_1+k_2+k_3+2s-1)\,!}{(k_1+k_2+k_3+s-1)\,!} \times \prod_{i=1}^{3} \frac{(2k_i)\,!}{k_i\,!(k_i-1)\,!}$$

String answer

$$\langle \mathcal{O}_+^3 \rangle_{g=0} \prod_{i=1}^{3} c_{+,0}^{2k_i} + \langle \mathcal{O}_-^3 \rangle_{g=0} \prod_{i=1}^{3} c_{-,0}^{2k_i} = \frac{\gamma^{2(k_1+k_2+k_3)-2}}{\sqrt{1-12t_4}} \prod_{i=1}^{3} \frac{(2k_i)\,!}{(k_i-1)\,!k_i\,!}$$

$$\gamma(t_4) = \frac{1-\sqrt{1-12t_4}}{6t_4}$$

Agreement to all orders in 't Hooft coupling!

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \,.\, T \,.\, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

= *Axiomatization of topological strings*

[Kontsevich,Manin ('94); Givental ('01,04,07); Teleman ('12); Pandharipande-Pixton-Zvonkine ('13)]

Coupling to Gravity

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Used in many previous contexts,

e.g. GW on $\mathbb{P}^1$ and its B-model LG mirror, r-spin class…

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

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« R-Matrix Action »

depends on x ~ universal

« Translation »

depends on y ~on V(M)

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \ldots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \, . \, T \, . \, \langle \mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Encodes degenerations of WS

depends on x ~ universal

Encodes choice of Ω

depends on y ~on V(M)

Coupling to Gravity

« CohFTs » as computational tools

General Correlator

$$\int_{\bar{\mathcal{M}}_{g,n}} \langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g \, \psi_1^{d_1} \dots \psi_n^{d_n}$$

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \, . \, T \, . \, \langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Fully Explicit

$$T^{\alpha}(u) = u - \sqrt{\frac{uH_{\alpha}}{2\pi}} \int_{\Gamma_\alpha} dy e^{-\frac{x - x(\alpha)}{u}}$$

+ similar for R-Matrix

Coupling to Gravity

« CohFTs » as computational tools

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R \, . \, T \, . \, \langle \mathcal{O}_{\alpha_1} \ldots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Dictionary:

$$T^\alpha(u) = u - \sqrt{\frac{uH_\alpha}{2\pi}} \int_{\Gamma_\alpha} dy e^{-\frac{x - x(\alpha)}{u}}$$

$$\Omega \leftrightarrow dy$$

$$W \leftrightarrow x$$

$$\int_{\Gamma^\alpha} \Omega e^{-\frac{W - W_\alpha}{2u}}$$

(Equivariant)
Worldsheet Disc Partition Function
[Nekrasov ('18)]

Coupling to Gravity

« CohFTs » as computational tools

CohFT

$$\langle\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle\rangle_g^{+top.grav} = R\,.\,T\,.\,\langle \mathcal{O}_{\alpha_1} \dots \mathcal{O}_{\alpha_n} \rangle_g^{pure\ matter}$$

Dictionary:

$$R^\alpha_\beta(u) = -\sqrt{\frac{uH_\alpha}{2\pi}} \int_{\Gamma_\alpha} e^{-\frac{W - W_\alpha}{u}} \theta_\beta$$

(Equivariant)
Worldsheet Disc One-Point Functions
with insertion of operator

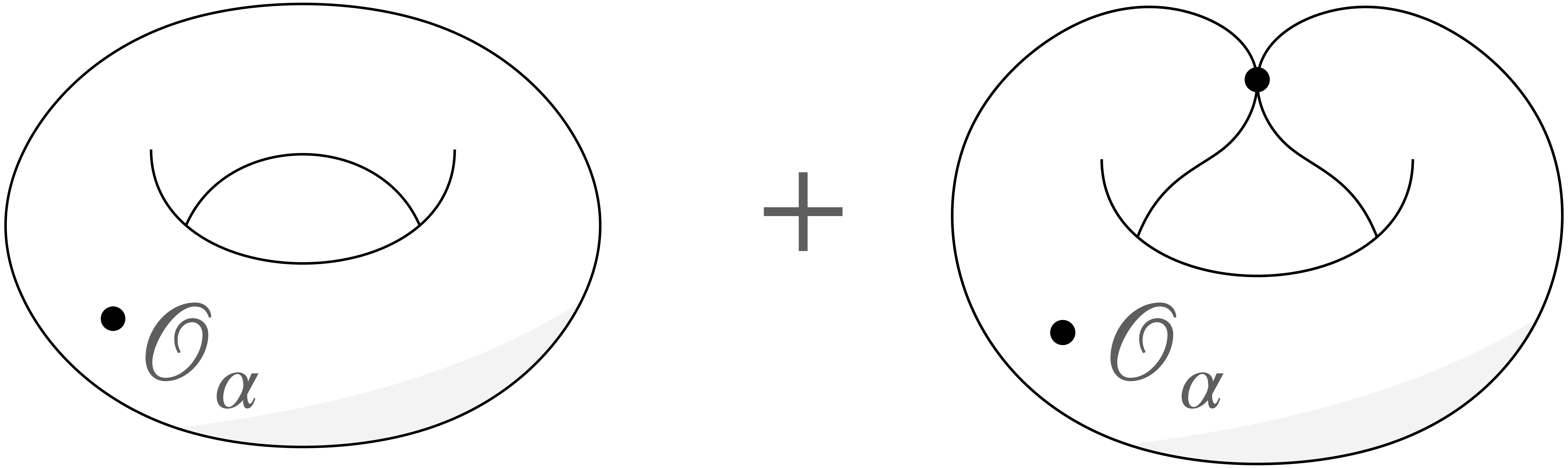
[Nekrasov ('18)]

Why these 2 objects suffice and are the right objects to compute
still not fully understood from worldsheet .

A Worldsheet Feynman Calculus

Example at genus one

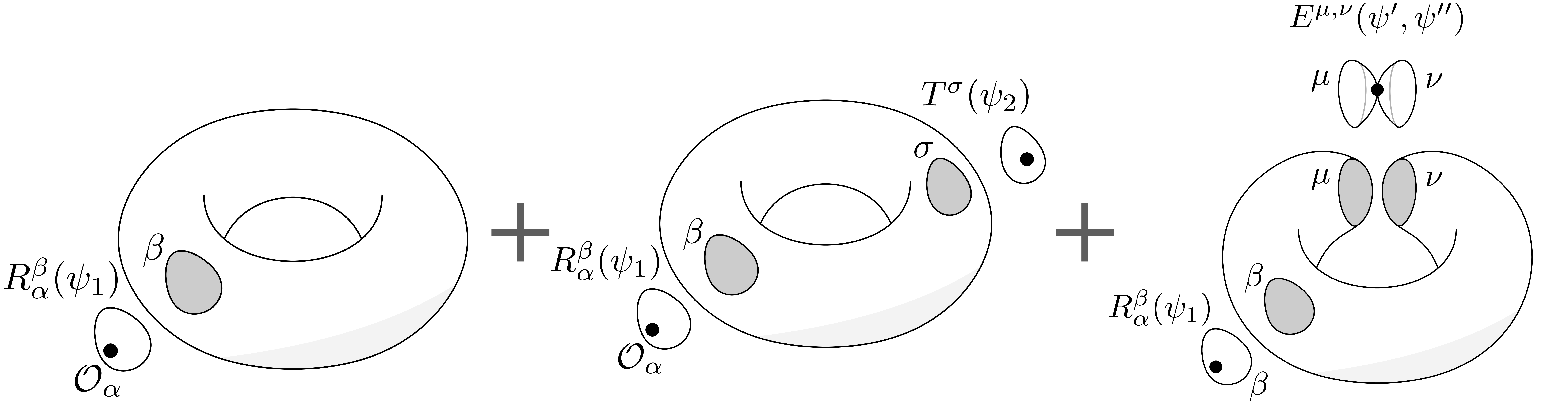
$$\langle\langle \mathcal{O}_\alpha \rangle\rangle_{g=1}^{CohFT}$$



Extra contribution from boundary of moduli space

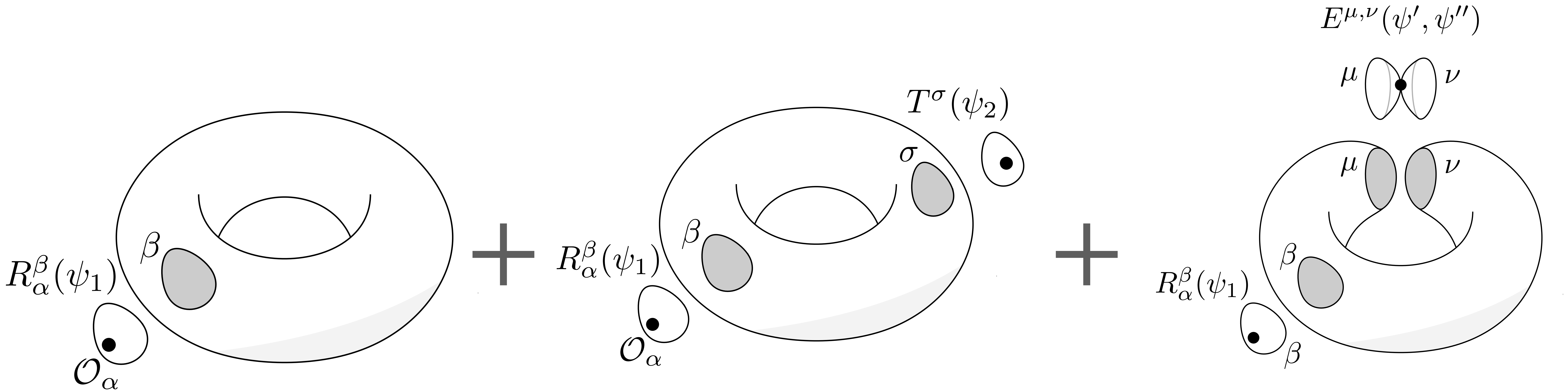
A Worldsheet Feynman Calculus

Example at genus one $\langle\langle \mathcal{O}_\alpha \rangle\rangle^{CohFT}_{g=1}$



A Worldsheet Feynman Calculus

Example at genus one $\langle \mathcal{O}_\alpha \rangle_{g=1}^{CohFT}$



$$\sum_{\beta\in\pm} R_\alpha^\beta(\psi)\langle \mathcal{O}_\beta \rangle_{g=1}^{TFT} + \sum_{\beta,\sigma\in\pm} R_\alpha^\beta(\psi)\langle \mathcal{O}_\beta \mathcal{O}_\sigma \rangle_{g=1}^{TFT} T^\sigma(\psi') + \sum_{\beta,\mu,\nu\in\pm} R_\alpha^\beta(\psi)\langle \mathcal{O}_\beta \mathcal{O}_\mu \mathcal{O}_\nu \rangle_{g=0}^{TFT} E^{\mu\nu}(\psi', \psi'')$$

Another MM Comparison

Example at genus one

String answer

$$\langle \mathscr{V}_{2k} \rangle_{g=1} = \sum_{\alpha\in\pm} c^{2k}_{\alpha,1} \langle \mathcal{O}_\alpha \rangle^{TFT}_{g=1} \int_{\bar{\mathscr{M}}_{1,1}} \psi_1 + c^{2k}_{0,\alpha} \int_{\bar{\mathscr{M}}_{1,1}} \langle\langle \mathcal{O}_\alpha \rangle\rangle^{CohFT}_{g=1}$$

= Matrix answer

$$\langle TrM^{2k} \rangle_{g=1} = \left(\frac{k-1}{12} - t_4 \gamma^2 \frac{k-2}{2} \right) \frac{\gamma^{2k-2}}{1-12t_4} \frac{(2k)!}{(k-1)!k!} .$$

(Agreement checked using Non-Pert. Matrix Schwinger-Dyson Eqs)

Sanity Check

Example at genus one

String answer

$$\langle TrM^{2k}\rangle_{g=1} = \left(\frac{k-1}{12} - t_4\gamma^2\frac{k-2}{2}\right)\frac{\gamma^{2k-2}}{1-12t_4}\frac{(2k)!}{(k-1)!k!}\,.$$

$$\Big\downarrow$$

Zero Coupling Limit

$t_4 \to 0 \quad (\gamma \to 1)$

[Harer,Zagier ('86)]

$$\langle TrM^{2k}\rangle_{g=1}^{GUE} = \frac{k-1}{12}\frac{(2k)!}{(k-1)!k!}$$

Reproduce (Free-field) Feynman Diagram Computation, gives Integers

Why the correlators always agree

CohFTs obey Topological Recursion

Spectral Curve
(x,y)

Matrix Model

$$\omega_{g,n} = \langle \prod_{i=1}^{n} Tr\left(\frac{dx(z_i)}{x(z_i) - M}\right)\rangle_g$$

=

Origin of
Operator Dictionary

String Theory

$$\omega_{g,n} = \int_{\bar{\mathcal{M}}_{g,n}} \langle \prod_{i=1}^{n} \sum_{\alpha_i \in \pm} \mathcal{O}_{\alpha_i} \rangle_g \sum_{d_i} \psi_i^{d_i} d\xi(z_i)_{\alpha_i, d_i}$$

[Chekov, Eynard, Orantin ('06); Eynard ('11,12); Dunin-Barkowski-Orantin-Shadrin-Spitz ('14)]

An Extreme (BMN-like) Limit

« Universal Subsector »

$$\lim_{\kappa\to\infty} \frac{\langle \prod_{i=1}^{n} \mathrm{Tr}\, M^{\kappa a_i} \rangle_g}{(2\gamma)^{\kappa |a|} \kappa^{3g-3+3n/2}} = \left(\frac{\langle \mathcal{O}_+^n \rangle_g^{LG}}{\Omega_+^n} + (-1)^{g-1} \frac{\langle \mathcal{O}_-^n \rangle_g^{LG}}{\Omega_-^n} \right) \times \frac{1}{(2\gamma)^{3g-3+n}} \sum_{m_1,\ldots,m_n \geq 0} \int_{\overline{\mathcal{M}}_{g,n}} \prod_{i=1}^{n} \frac{\psi_i^{m_i} a_i^{m_i+\frac{1}{2}}}{\sqrt{2\pi}}$$

Reflects Airy Universality
(dominated by eigenvalues close to edge)

Reproduce + Generalize Okounkov-Pandharipande (GUE)!

An Extreme (BMN-like) Limit

« Decoupling the Matter Theory »

$$\lim_{\kappa\to\infty} \frac{\langle \prod_{i=1}^n \mathrm{Tr}\, M^{\kappa a_i}\rangle_g}{(2\gamma)^{\kappa|a|}\kappa^{3g-3+3n/2}} = \left(\frac{\langle \mathcal{O}_+^n \rangle_g^{TFT}}{\Omega_+^n} + (-1)^{g-1} \frac{\langle \mathcal{O}_-^n \rangle_g^{TFT}}{\Omega_-^n} \right) \times \frac{1}{(2\gamma)^{3g-3+n}} \sum_{m_1,\dots,m_n\geq 0} \int_{\overline{\mathcal{M}}_{g,n}} \prod_{i=1}^n \frac{\psi_i^{m_i} a_i^{m_i+\frac{1}{2}}}{\sqrt{2\pi}}$$

From our operator dictionary

$$\lim_{k\to\infty} Tr\, M^{2k} \sim \mathcal{O}_\alpha \times 2^{2k} \sum_d k^{d+1/2} \psi^d$$

Maximize ψ-class insertions

$\leftrightarrow$

Decouples matter theory

$$\langle O_{\alpha_1} \dots O_{\alpha_n} \rangle^{TFT} \times \int_{\mathcal{M}_{g,n}} \prod_{i=1}^n \sum_d c_{\mathbf{k_i}>>1,d}\ \psi_i^d$$

Pure 2d top gravity in BMN limit!

Summary: a String Dual for the 1-Matrix Model

- Standard ‘t Hooft limit, no double scaling
- Explicit string dual is B-twisted LG model coupled to 2d topological gravity
- Spectral curve dictates worldsheet string action: $x(z) = W_{LG}(z)$ & $\Omega = dy$
- Precise dictionary between sing trace « operators » and vertex operators on the worldsheet: $TrM^k \leftrightarrow \sum_{\alpha \in +,-} \sum_{d=0}^{k-1} c^k_{\alpha,d} \, \mathcal{O}_\alpha \psi^d$
- CohFT technology: reduce to explicit, computable integrals over $\bar{\mathcal{M}}_{g,n}$!
- Keeps track of subtle contact terms/degenerations of worldsheet
- Correlators agree to all orders in 1/N and all orders in ‘t Hooft coupling(s)

Thank You!

https://cocalc.com/agiacche/ws-duals-1-mm/demo

A New Perspective on the Double-Scaling Limit

What we were zooming into if the string theory already existed?

$$\Omega = dy \to 0$$

where $dW = dx = 0$

Geometry of Criticality

$$t_4 \to t_c = \frac{1}{12}$$

Cusp singularites at critical points!

Strings on Full Geometry

$$W = \gamma(t_4)\left(\phi + \phi^{-1}\right)$$

$$\Omega = \gamma\left(1 - 3t_4\gamma^2(1+\phi^2)\right)d\phi$$

$$g_s = \frac{1}{N}$$

DSL

$$\epsilon \to 0,\ N \to \infty$$

$$t_4 = \frac{1-\epsilon^2}{12}$$

$$\phi = 1 + \sqrt{\frac{\epsilon}{2}}\tilde{\phi} + O(\epsilon)$$

Strings on Blow-Up of Singularity

$$\tilde{W} = \frac{1}{\sqrt{2}}(\tilde{\phi}^2 - 2)$$

$$\tilde{\Omega} = (\tilde{\phi}^2 - 1)d\tilde{\phi}$$

$$\tilde{g}_s = \frac{1}{N(t_4 - t_c)^{5/4}}$$

Outlook for deriving AdS/CFT

Next Step: chiral algebra of $\mathcal{N} = 4$ SYM